Disability with partial insurance in production economies

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Abstract

We extend the standard Ramsey model by a precautionary saving motive, and examine the economy with the risk of permanent loss of income. We show that the optimal behavior of disabled individuals leads them to poverty, which results from incomplete insurance against disability and from the general equilibrium features of the model. We calibrate the model to study how different insurance schemes impact welfare and the aggregate economy. We show that moving from the current system in the U.S. towards a private insurance system increases welfare. However, some type of insurance is needed, since pure self-insurance would reduce welfare substantially.

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1 Introduction

This paper discusses the effects of uncertainty associated with rare but permanent events, such as severe injury or compulsory retirement. Generally, we will consider these types of events as an onset to disability. Although the probability of disability is rather small for working-age people in the United States, the risk is great, since it may lead to permanent loss of labor income.[1] We will analyze how this type of shock – a shocks with permanent transition – impacts the behavior of individuals, and the economy as a whole, in the standard incomplete markets model type of Aiyagari (1994).

First, we ask how large the loss of welfare is due to incomplete insurance against disability, to get an idea of the economic importance of the issue. In addition to quantitative analyzes, our model also enables a transparent way to study the mechanisms that cause welfare loss. Second, we explore how the welfare loss might be reduced by studying the performance of different insurance schemes against disability. This is important since Social Security Disability Insurance (SSDI) is one of the largest social insurance programs in the United States and the growth of the program in size and expense during the last two decades has been significant, causing substantial fiscal ramifications for the federal budget.[2] The first alternative insurance system that we consider is a private one. We ask if it would be possible to move from the current system to a private one, such that everyone's welfare would increase. This is not clear, since an increase in private transfers reduces the welfare of workers who have to finance private insurance, whose coverage across the population is assumed to be better than SSDI. Second, we consider pure self-insurance, which provides good insurance against moderate fluctuations in labor income, as shown by Aiyagari (1994).

[1] Bound and Burkhauser (1999, Table 2) summarize evidence of self-reported disability status from five different data sets. For working-age males, the percentage of the population with disabilities varies from 8.1% to 11.7%. For women, the corresponding numbers are 7.8% and 11.6%. The size of the disability risk is analyzed by Meyer and Mok (2008), who report that ten years after severe disability onset, a person experiences on average a 68% decline in earnings. Stephens (2001) and Low and Pistaferri (2010) also report significant decline in earnings due to onset of disability.

[2] In 2009, the program covered almost 7.8 million disabled workers, and its costs were $118 billion (SSA, 2010). Autor and Duggan (2006), Duggan and Imberman (2009) and Milligan (Forthcoming) provide good overviews and discussions on sources of the growth for SSDI.

To address households' consumption and labor supply with partial insurance, we extend the standard, continuous time Ramsey model by including a precautionary saving motive. That is, households self-insurance by saving against the risk of disability, and this additional saving can be seen as a measure for aggregate precautionary savings (Huggett and Ospina, 2001). In addition to self-insurance, both private and social insurance against disability exist. We quantify the share of private risk sharing, but the source of this insurance is exogenous, in the tradition of standard incomplete markets model. The government also provides social insurance financed by a proportional tax rate, which distorts economic behavior.

We show that completing the insurance markets would increase welfare in the economy by about 0.4% in consumption equivalent units, and aggregate output and consumption would increase by about 1.5%. Therefore, policy reforms that improve insurance against disability could also increase welfare and the performance of the aggregate economy quantitatively significantly. Consequently, we are able to show that moving from the current system to a private one could lead to Pareto improvement. In the case of pure self-insurance 38% of the capital stock would be hold for precautionary saving reasons which shows that potentially the risk of disability is an important source of income uncertainty. However, we emphasize that self-insurance works poorly against permanent shocks when private risk sharing is likely.

The key mechanism behind the results is driven by endogenous prices and the general equilibrium feature of the model. The incomplete insurance against disability causes a precautionary saving motive for workers, which lowers the interest rate. Since disabled agents do not have income uncertainty, the existing interest rate in the economy is "too low" for them. As a result, they will have a declining path for consumption. The declining path for consumption lowers the expected consumption in the disability state, which increases the precautionary saving motive of workers, which in turn lowers the interest rate, and so forth. The driving force behind this outcome is that agents do not take into account how their decisions influence prices. This type of effect is sometimes referred as a "pecuniary externality".

This paper contributes to two branches of literature. First, the model introduced in this paper gives a new extension for the standard textbook Ramsey model, and builds a bridge between the continuous time Ramsey models and discrete time incomplete markets models. Generally, the effects of uninsurable idiosyncratic shocks on individual and aggregate outcomes in dynamic general equilibrium is a much studied issue.[3] However, the literature is more focused on moderate fluctuations of uncertain labor income, whereas we focus on the uncertainty associated with permanent loss of labor income. Therefore, this model is new, and fills a gap in the existing literature. Moreover, the model in this paper is more transparent and tractable than discrete time models with precautionary savings. For instance, we can analyze the system of differential equations that characterize the aggregate economy by linearizing the system around the steady state. Closest to our model is the tractable model of precautionary saving by Toche (2005). We expand this partial equilibrium model to a general equilibrium.[4]

The second branch of literature associated with this paper is the economics of disability, and the analysis of disability insurance targeted to disabled people.[5] The existing literature associated with this paper has focused on the insurance-incentive-problem related to the SSDI program (see Golosov and Tsyvinski 2006, 2007 and Low and Pistaferri 2010, for example).[6] This paper contributes to this literature by using a growth model, and by studying the effects of disability within a dynamic general equilibrium framework. We show that the effects arising from the general equilibrium setup of the model causes behavior that should be taken into account when different insurance schemes against disability are analyzed.

The rest of the paper is organized as follows: Section 2 introduces the model. Section 3 analyzes the dynamics of the economy in equilibrium. Section 4 discusses the calibration of the model and reports the numerical results of simulations. Section 5 concludes the paper. The Web Appendix

[3]The seminal articles are Huggett (1993) and Aiyagari (1994), and literature is reviewed by Heathcote, Storesletten, and Violante (2009b) and Guvenen (2011).

[4]See, also the recent papers by Carroll and Toche (2009) and Carroll and Jeanne (2009). Another model close to the model in this paper is Heathcote, Storesletten, and Violante (2009a), in which a tractable model with partial insurance is developed. However, they focus on an economy where households trade bonds, but here the only asset is productive capital.

[5]A literature survey is provided by Bound and Burkhauser (1999).

[6]Diamond and Sheshinski (1995) have also modeled disability with a static model, and Diamond and Mirrlees (1978) and Chandra and Samwick (2009) have modeled the economic effects of the risk of disability within a partial equilibrium setting. There is also a large body of empirical literature that tries to estimate the “genuine” number of disabled persons, or how SSDI matter for labor supply: See, for example, Parsons (1980); Bound (1989); Gruber (2000); Black, Daniel, and Sanders (2002); Benitez-Silva, Buchinsky, and Rust (2004); Chena and van der Klaauw (2008). Further, a significant amount of papers try to estimate the size of different factors behind the growth in SSDI cost: See especially, Autor and Duggan (2003), Duggan and Imberman (2009) and references therein.

associated with this paper provides details about the model, discusses extensions of the model and reports the robustness of the results.

2 The model

The model introduced in this paper is a standard one-sector neoclassical growth model, or the Ramsey model in continuous time with an endogenous labor supply, but there are two significant exceptions compared to the baseline model. First, each employed member in each household faces the constant probability of losing her job permanently throughout her life. Second, the insurance markets against this uncertainty are incomplete, but there is partial risk sharing via lump sum transfers and the government provides insurance against disability, which is financed by a proportional labor income tax. Since both insurances against uncertainty are only partial, the members of households have a precautionary saving motive and they can self-insure by holding a single asset – physical capital. However, the effects of precautionary savings on output are ambiguous, since a higher capital stock implies a wealth effect that reduces labor supply, as pointed out by Marcet, Obiols-Homs, and Weil (2007).

2.1 Demographics and disability

The economy consists of a continuum of households of measure one and every household consists of a large number of members. At every moment, the economy receives a large number of new members with size nL_0e^{nt}, where L_0 is normalized to 1, when the size of the population at time t is $P_t = \int_{-\infty}^{t} ne^{nj}dj = e^{nt}$. The new members are equally distributed to existing households and are born to work, but they may face a disability shock. That is, every instant there is the constant instantaneous probability μ of permanent disability for each employed member in each household. The transition from the employed state to the disability state follows a Poisson process with arrival rate μ. We assume that the law of large numbers holds when μ is also a fraction of workers who face permanent disability at each instant. Therefore, the size of employed population is given by $L_t = \int_{-\infty}^{t} ne^{nj}e^{-\mu(t-j)}dj = \theta e^{nt}$, where $\theta \equiv \frac{n}{n+\mu}$.[7] A fraction θP_t of the total population is workers, when the fraction of disabled population is $D_t = P_t - L_t = \lambda e^{nt}$, where $\lambda \equiv \frac{\mu}{n+\mu}$.

2.2 Production and factor prices

The Cobb-Douglas production function is assumed with constant returns to scale and there exists a perfect competition in factor and production markets. Hence, the economy's production per efficient capita Y_t is given by

$$Y_t = K_t^{\alpha} N_t^{1-\alpha}, \tag{1}$$

where K_t is the aggregate capital stock, α is the capital share and N_t is aggregate labor.[8]

Profit maximization of the representative firm gives following factor prices:

$$r_t = \alpha K_t^{\alpha-1} N_t^{1-\alpha} - \delta, \tag{2}$$

$$w_t = (1-\alpha) K_t^{\alpha} N_t^{-\alpha}, \tag{3}$$

where δ is the depreciation rate of capital stock, r_t is the prevailing interest rate and w_t is the wage rate.

2.3 Government

We assume that the evaluation process of SSDI applications is incomplete. That is, for any application for SSDI there is a probability π that the application is approved. Hence, SSDI evaluators deny benefits to some individuals who should have it. For the members of households who are covered by SSDI, the government offers social insurance, which is a lump sum transfer,

[7] We assume that the structure of population is always at its steady-state distribution.

[8] Production per efficient capita means that we have scaled the actual production by the term $P_t Z_t$, where Z_t describes technological progress, for which the growth rate is g. Moreover, our Cobb-Douglas production function is in a Harrod-neutral form, where effective labor is defined by $Z_t l_t L_t$, where l_t is the labor supply of the representative worker. Hence, in equilibrium $N_t = \theta l_t$.

b_t^g, for every period. To finance this program, the government must lay a labor income tax, τ_t, and the government is running a balanced budget for every period, or a pay-as-you-go social insurance program.[9] This implies the following budget constraint:

$$B_t = \tau_t w_t N_t. \tag{4}$$

The left-hand side of equation (4) shows the aggregate transfer for the disabled members of households when $B_t = b_t^g \pi\lambda$, where $\pi\lambda$ is the fraction of people in SSDI. The right-hand side of equation (4) shows the net income of the government, which is composed of the taxed labor income.

It is assumed that the government sets a rate η, which is the replacement ratio of labor income when $0 \leq \eta \leq 1$. Therefore, the level of social insurance for each disabled member is defined as $b_t^g = \eta w_t l_t$, where l_t is the labor supply of representative employed member. The tax rate is an endogenous variable, and only the replacement ratio η is decided by the government. Rewriting the government budget constraint (4), we can define the tax rate by $\tau_t = \eta\pi\lambda/\theta$.

2.4 The problem of the representative household

The representative household starts its living in the economy at $t = 0$, acting like maximizing its current members' and their prospective descendants' expected utility by making consumption and labor supply decisions. The household does not know how many of its members will be disabled at each instant, but it knows the probability for permanent disability for its employed members. Since the transition from the employed state to the disability state follows a Poisson process, the probability is the same for each employed member of the household. We also assume that the new members of the household are "loved", when newborn members enter the economy with an amount of assets equal to the asset holdings of those currently employed.

[9]The SSDI program is basically financed with a payroll tax, but it is also possible to set taxes on both labor and capital incomes. Web Appendix B discusses this extension and shows the results from the model where capital income is taxed in addition to labor income. However, the results are almost the same as in this specification.

The problem of the household is given by the following equations:

$$\max_{(c_t>0,0\leq l_t\leq 1)_{t\in[0,\infty)}} \quad E_0 U = E_0 \int_0^\infty e^{(n-\rho)t}\left(\log c_t - \gamma \frac{l_t^{1+\phi}}{1+\phi}\right) dt \tag{5}$$

$$\text{s.t.} \quad \dot{a}_t = (r_t - n - g)\, a_t - c_t + (1-\epsilon_t)\left[(1-\tau_t) w_t l_t - b^p\right] + \epsilon_t b_t \tag{6}$$

$$\lim_{t\to\infty}\left[a_t e^{-\int_0^t r_s - g - n ds}\right] = 0 \tag{7}$$

and a_0 given.

E_0 is the conditional expectation operator and the household gets utility from consumption per adult person c_t and the supply of labor l_t produces disutility for the household. However, in the disabled state, $l_t = 0\ \forall\ t$ by the definition of disability.[10] n is the growth rate of members in the household, $\rho > n$ is the rate of time preference, g is the growth rate of productivity and variables are scaled per efficient capita. Frisch elasticity is equal to $\frac{1}{\phi}$.

Disability is described by $\epsilon_t \in \{0,1\}$, which follows a Poisson process for the employed members of the household. They have never been disabled ($\epsilon_v = 0\ \forall\ v \leq t$), but an occurrence of a disability happens with a probability μ at every instant. However, an employed member who has been disabled in the past ($\epsilon_v = 1$ for some $v \leq t$), will remain disabled forever: $\epsilon_t = 1\ \forall\ t$ without any uncertainty.

The household faces a budget constraint, where r_t is the interest rate and w_t is the wage rate. When its members are employed, they receive capital income from assets holdings a_t and labor income from working, which is taxed at rate τ_t by the government. Further, there is a risk sharing arrangement: The employed members of the household must give a transfer size of b^p to the disabled members of the household. Therefore, the expected income for the disabled members of the household is composed of capital income and tax free lump sum transfers $b_t \equiv \pi b_t^g + b$. A member who is disabled will receive SSDI benefits, b_t^g, with probability π, but will get a private

[10] We assume that leisure and consumption are separable, ensuring that the marginal utility for consumption is higher in the disability than employed state, which is needed for a precautionary saving motive. Separability between consumption and leisure, in turn, implies a log-utility over consumption, as shown by King, Plosser, and Rebelo (1988), since we want to have the balanced growth path. Moreover, we could include a constant in the utility function to capture the value of leisure in the disability state, and the constant could be calibrated such that the utility of employed members is always higher than disabled ones. However, disability is not a choice variable here when this would not alter the results, and the constant is not included.

transfer, b, in any case.

We can solve the problem by using the familiar tools of optimal control theory, for which Toche (2005) provided the following key insight: The only source of uncertainty in this problem is the timing of transition from the employment state to the disability state, and after the transition, the problem is deterministic. That is, disabled members of the household do not face any kind of uncertainty. The key assumption is the persistence of this transition. Therefore, it is possible to solve the full problem by using "backward induction". First, solve the deterministic problem of the disabled members of the household, and then use that solution when solving the problem of employed members.[11] Web Appendix A.1 discusses in detail how the problem of the household is solved.

2.4.1 Euler equations and precautionary saving

Euler equations for consumption are:

$$\frac{\dot{c}_t^e}{c_t^e} = r_t - \rho - g + \mu\left(\frac{c_t^e}{c_t^d} - 1\right), \tag{8}$$

$$\frac{\dot{c}_t^d}{c_t^d} = r_t - \rho - g, \tag{9}$$

where $\dot{x}_t \equiv \frac{dx_t}{dt}$ and superscripts index the state, e stands for the employed state ($\epsilon = 0$), and d stands for the disabled state ($\epsilon = 1$). In the disabled state, the level of consumption follows Euler equation (9), which is in the standard form. However, in the employed state, the consumption behavior is affected by a precautionary saving motive, given by the last term in equation (8), which gives the expected loss of consumption due to disability. Note that the precautionary saving motive disappears when $\mu = 0$ or $c_t^e = c_t^d$. That is, if the probability of a permanent transition to the disability state is zero, or if the levels of consumption are the same in both states when there is a perfect insurance against uncertainty, the precautionary saving motive disappears. The higher the μ or the greater the difference in the levels of consumption between the states (i.e., the lower the level of insurance), the higher the precautionary saving motive.

[11] For a more detailed discussion, see Toche (2005), Carroll and Kimball (2007) and Carroll and Toche (2009).

The household must also decide its labor supply for its employed members. The first order condition for labor supply is given by

$$\gamma l_t^{\phi} = \frac{(1-\tau_t)w_t}{c_t^e}. \tag{10}$$

That is, the marginal disutility from working must be equal to the proportion of after-tax wage to consumption, which is the standard result. We have now derived Euler equations for the problem of the household, but we must still define the value of c_t^d. We can do this by solving the problem of disabled members, and then derive the level of consumption for the representative member in the disability state, c_t^d.

2.4.2 The behavior of disabled members of the household

The level of consumption of a disabled member depends on how long she has been in the disability state.[12] Let $a_{j,v}^d$ and $c_{j,v}^d$ be the consumption and asset at time v for a member of the household who was disabled at time $j \leq v$. The current time is t. The budget constraint for this type of member can be written as follows:

$$\begin{aligned} \dot{a}_{j,v}^d &= (r_v - n - g)\, a_{j,v}^d - c_{j,v}^d + b_v \\ a_{j,j}^d &= a_j^e. \end{aligned} \tag{11}$$

Now, the modification is that the assets and consumption are a function of j. Moreover, $b_v \equiv \mathbf{I} b_v^g + b$, where $\mathbf{I}$ is an indicator function that takes the value of one when a disabled member receives SSDI benefits, or zero otherwise.

The consumption function can be solved by using Euler equation (9) and straightforward integration combined with equation (11) gives

$$c_{j,t}^d = (\rho - n)(a_{j,t}^d + \tilde{b}_t), \tag{12}$$

[12] This part of the model builds on the overlapping generation models by Blanchard (1985), Buiter (1988) and Weil (1989).

where $\tilde{b}_t = \int_t^\infty b_v e^{-(\bar{r}_{v,t}-n-g)(v-t)} dv$ and $\bar{r}_{t,v} = \frac{1}{v-t}\int_t^v r_s ds$. That is, $\tilde{b}_t$ is the present value for the insurance income.

Euler equations (8) and (9) imply that disabled members' consumption is not constant, since the interest rate and the capital stock are mainly set by the decisions of the workers.[13] The Euler equation of workers (8) includes the term $\mu\left(c_t^e/c_t^d - 1\right)$, which gives the precautionary saving motive and lowers the level of r_t. However, disabled members face the same interest rate r_t in their Euler equation, which implies that $r_t < \rho + g$, and this causes the dis-saving of disabled members (or a descending path for consumption). The cause of continuous dis-saving is that the assets of the disabled member will approach asymptotically to the natural borrowing constraint.[14]

Note that the dis-saving of disabled members and the saving of employed ones are results of the general equilibrium setup of the model. We could say that disabled members are "impatient". That is, the interest rate is lower than their efficient discount rate, which causes a descending time path for consumption. However, "impatience" does not originate from the higher subjective discount factor compared to employed members, but rather from the fact that disabled members do not have a precautionary saving motive, or that they do not face any income uncertainty. Therefore, the prevailing interest rate in the economy – set by members that have a precautionary saving motive – causes the dis-saving of disabled individuals, or their "impatience".[15]

An important feature of this phenomenon is that households do not take into account the effects of their decisions on prices, and how changes in prices, in turn, affect other individuals' behavior. This externality is discussed in detail by Davila, Hong, Krusell, and Ríos-Rull (2005). Incomplete insurance against disability causes a negative externality, that is, the declining path of consumption for disabled persons, which is a well documented empirical fact by Meyer and Mok (2008) and Stephens (2001). We showed that this behavior may be optimal.[16]

[13]It is assumed that most people work and that only a small number of people are disabled. Further, the employed members of the households are richer than the disabled ones, and therefore, more influential when the aggregate economy is considered.

[14]We could also include a borrowing constraint $a_t \geq 0 \; \forall \; t$, but then only a steady state analysis would be possible. Web Appendix C introduces this extension to the model and compares steady state values under different borrowing constraints. It is shown that quantitatively, the effects of more strict borrowing constraint are small.

[15]The key assumption here is that the per capita social insurance benefit grows at the rate of g. Therefore, disabled members also enjoy the benefits of growing productivity.

[16]In this case, precautionary savings cause excess capital accumulation, which can be removed by setting a capital

2.4.3 The representative disabled member of the household

The household needs to know c_t^d, which is the level of consumption of the representative (or average) member in the disability state, in order to define the magnitude of precautionary savings, as indicated by equation (8). In the disability state, however, members' levels of consumption differ from each other depending on how long they have been disabled (i.e., depending on j), and whether or not they receive SSDI benefits. We can derive the aggregate consumption in the disability state by integrating equation (12) over j, but it must be noted that the size of members in the household is growing. Therefore, the representative member's consumption in the disability state is given by

$$c_t^d \equiv \int_{-\infty}^{t} n c_{j,t}^d e^{-n(t-j)} dj = (\rho - n)\left(a_t^d + \tilde{b}_t\right), \tag{13}$$

where we have defined that $a_t^d \equiv \int_{-\infty}^{t} n a_{j,t}^d e^{-n(t-j)} dj$, which is the asset holdings of the representative disabled member.

To derive the law of motion for a_t^d, we differentiate it in respect to time. This gives

$$\dot{a}_t^d = (r_t - n - g)\, a_t^d - c_t^d + b_t + n(a_t^e - a_t^d). \tag{14}$$

Equation (14) reproduces the results of equation (11), but the last term in the equation reflects the fact that asset accumulation is increased by the arrival at each instant of new members in the disability state, who have higher wealth than the average member in that state (i.e., $a_t^e > a_t^d$). More details can be found in Web Appendix A.2.

2.5 Aggregate economy

Variables, which define the behavior of the representative household, are scaled by technology and population. Multiplying these variables by the share of population associated with the state

tax for workers equal to $\mu\left(c_t^e/c_t^d - 1\right)$. Then, Euler equation in both states would be the same when dis-saving of disabled individuals would vanish and their consumption would be constant. The aggregate economy could then also be studied simply by using a standard Ramsey model. The model therefore makes a case for non-linear capital income taxation. Further examination of this issue is left to future studies.

in question defines the aggregate variables, since there is a continuum of households of measure one and population is always at its steady-state distribution. This implies that at the level of aggregate economy, the shares of employed and disabled population are constant. That is, at the aggregate level, it seems that there is no transition between the two states. However, significant dynamics are taking place at the household level due to idiosyncratic uncertainty.

2.5.1 Aggregate consumption and Euler equation

To derive Euler equation for aggregate consumption, we first define the aggregate consumption of the employed population, which is given by θc_t^e. Equation (13) defines the aggregate consumption of the disabled population when multiplied by λ. Hence, the aggregate consumption in per efficient capita, C_t, can be defined as $C_t = \theta c_t^e + \lambda c_t^d$.

In order to derive the aggregate Euler equation for consumption, we must differentiate C_t in respect to t, which yields $\dot{C}_t = \theta \dot{c}_t^e + \lambda \dot{c}_t^d$. Since the population is always at its steady-state distribution, Euler equations (8) and (9) determine the aggregate Euler equation for consumption, when these equations are multiplied by the share of population associated with the state in question. Therefore, the aggregate Euler equation for consumption can be written as

$$\dot{C}_t = (r_t - \rho - g) C_t + \frac{\mu}{\theta} \left(\frac{C_t}{c_t^d} - 1 \right) \left(C_t - \lambda c_t^d \right), \tag{15}$$

where we have used the determination of C_t to replace c_t^e. The first term describes the evolution of consumption without market incompleteness, and the second term captures the effects of precautionary saving. Note that if $\mu = 0$ or $c_t^d = c_t^e = C_t$, Euler equation for consumption (15) is equal to the Euler equation in the standard Ramsey model.

2.5.2 The evolution of stock of assets

The asset holdings of households can be aggregate in the same manner as consumption. That is, $A_t = \theta a_t^e + \lambda a_t^d$, where equation (14) defines the aggregate asset holdings of the disabled population when it is multiplied by λ. The evolution equation for the aggregate asset holdings of households, $\dot{A}_t$, can be aggregate directly from equation (6) by rewriting it for both states separately, as follows:

$$\dot{a}_t \quad = \quad \begin{cases} (r_t - n - g)\, a_t - c_t + (1 - \tau_t) w_t l_t - b^p, & \text{if} \quad \epsilon = 1 \\ (r_t - n - g)\, a_t - c_t + b_t, & \text{if} \quad \epsilon = 0. \end{cases} \tag{16}$$

The evolution of stock of assets can now be derived by using equation (16) and from the following facts: The population is always at its steady-state distribution, the government was running a balanced budged and all output is exhausted to households. That is, by using equations (1), (2), (3) and (4) we can write the evolution equation as follow:

$$\dot{A}_t = Y_t - C_t - (\delta + g + n) A_t \tag{17}$$

and it is the standard one.

3 The dynamics of the economy in equilibrium

The dynamics of the economy can now be described by the system of four differential equations: the Euler equation for aggregate consumption (15); the evolution equation for the aggregate stock of assets (17); the equation for the growth of labor supply (see equation (21) below); and the evolution equation for the assets holdings of the disabled members of households (14). Since the Euler equation for aggregate consumption depends on c_t^d, which is a function of a_t^d, we also need the evolution equation for a_t^d, in addition to the standard equations.

3.1 Equilibrium

The equilibrium in the model is as follows:

Definition 1. *A sequential markets equilibrium consists of allocation for the representative household* $(c_t, l_t, a_t)_{t\in[0,\infty)}$*, allocation for the firm* $(K_t, N_t)_{t\in[0,\infty)}$*, government policy* $(b_t^g, \tau_t)_{t\in[0,\infty)}$*, prices* $(r_t, w_t)_{t\in[0,\infty)}$ *and aggregate allocations* $(C_t, A_t, B_t)_{t\in[0,\infty)}$*, such that:*

1. ***The representative household optimizes**: Given prices $(r_t, w_t)_{t\in[0,\infty)}$, government policy $(b^g_t, \tau_t)_{t\in[0,\infty)}$, a lump sum transfer b^p and b and a_0, the allocation $(c_t, l_t, a_t)_{t\in[0,\infty)}$ maximizes (5) subject to (6), $c_t > 0$ and $0 \le l_t \le 1$ for all t and (7).*

2. ***The firm optimizes**: Given prices $(r_t, w_t)_{t\in[0,\infty)}$, the allocation $(K_t, N_t)_{t\in[0,\infty)}$ solves (2) and (3).*

3. ***Government satisfies its budged constraint**: Given η, π, prices $(w_t)_{t\in[0,\infty)}$ and allocation $(N_t)_{t\in[0,\infty)}$, government policy $(b^g_t, \tau_t)_{t\in[0,\infty)}$ satisfies (4).*

4. ***Consistency condition between aggregate and individual behavior**: The aggregate consumption is given by $C_t = \theta c^e_t + \lambda c^d_t$, the aggregate asset holdings are given by $A_t = \theta a^e_t + \lambda a^d_t$, the aggregate supply of labor is given by θl_t, the aggregate transfer for disabled population by the government, B_t, is given by (4) and the law of motions (14), (15) and (17) are consistent with individual behavior for all t.*

5. ***Markets clear**: The labor markets clear: $N_t = \theta l_t$, the asset markets clear: $K_t = \theta k^e_t + \lambda k^d_t = \theta a^e_t + \lambda a^d_t = A_t$ and the goods markets clear: $\dot{K}_t = Y_t - C_t - (\delta + g + n)K_t$, where Y_t is given by (1) for all t.*

Now, the dynamics of the economy in the equilibrium can be described by the following system of equations:

$$\dot{\hat{C}}_t = (r_t - \rho - g)\,\hat{C}_t + \frac{\mu}{\theta}\left(\frac{\hat{C}_t}{\hat{c}^d_t} - 1\right)\left(\hat{C}_t - \lambda\hat{c}^d_t\right) - \frac{\dot{l}_t}{l_t}\hat{C}_t \tag{18}$$

$$\dot{\hat{K}}_t = \hat{Y}_t - \hat{C}_t - (\delta + g + n)\hat{K}_t - \frac{\dot{l}_t}{l_t}\hat{K}_t \tag{19}$$

$$\dot{\hat{k}}^d_t = (r_t - n - g)\,\hat{k}^d_t - \hat{c}^d_t + \hat{b}_t + (n + \mu)(\hat{K}_t - \hat{k}^d_t) - \frac{\dot{l}_t}{l_t}\hat{k}^d_t. \tag{20}$$

The hat indicates that variables are also divided by l_t.[17] Moreover, the growth rate for the labor

[17] The rest of variables are given by the following equations: r_t is given by equation (2), $\hat{c}^d_t$ is given by equation (13) and $\hat{b}_t = \pi\eta\hat{w}_t + \hat{b}$. Finally, we use equations (1) and (3) to substitute $\hat{w}_t$ and $\hat{Y}_t$ into previous definitions.

supply can be derived from equation (10), and by using equations (8) and (19) we get

$$\frac{\dot{l}_t}{l_t} = \frac{\alpha}{\alpha+\phi}\left[\frac{\hat{Y}_t}{\hat{K}_t} - \frac{\hat{C}_t}{\hat{K}_t} - (\delta+g+n)\right] - \frac{1}{\alpha+\phi}\left[r_t - g - \rho + \frac{\mu}{\theta}\left(\frac{\hat{C}_t}{\hat{c}_t^d} - 1\right)\right]. \tag{21}$$

Substituting equation (21) into equations (18) to (20) gives us three equations and three unknown variables: $\hat{C}$, $\hat{K}$ and $\hat{k}^d$.[18]

3.2 The steady state and the uniqueness of the equilibrium

We now define the system of equations (18), (19) and (20) as $\dot{\mathbf{x}}_t = F(\mathbf{x}_t)$, where $\mathbf{x} = (\hat{K}, \hat{k}^d, \hat{C})$ denotes coordinates in the phase space and F represent a smooth map that describes the evolution of the dynamic system. The steady state implies that $\dot{\mathbf{x}}_t = 0$, and the steady state values of $\mathbf{x}_\star$ can be numerically solved.[19]

The stability of the system can be locally analyzed by taking a first-order Taylor approximation around the steady state, when the real part of eigenvalues ν of the Jacobian matrix determine the nature of the steady state. By applying this procedure to our system of differential equations, we can write our differential equations as $\dot{\mathbf{x}}_t \approx \mathbf{A}(\mathbf{x}_t - \mathbf{x}_\star)$, where $\mathbf{A} = DF(\mathbf{x}_\star)$ is the Jacobian matrix, and the eigenvalues of the Jacobian are $\nu_1 > 0$, $\nu_2 < 0$ and $\nu_3 < 0$. Thus, the Grobman-Hartman Theorem implies that we have a locally saddle path stable steady state. This also implies that the equilibrium is (locally) unique.

4 Simulations

In simulations we address three issues: i) How large a welfare loss results from incomplete insurance against disability; ii) How different insurance schemes impact welfare and the aggregate

[18]The precautionary saving motive for the working members of households causes that $\dot{l}_t/l_t \neq 0$ at the steady state. However, the baseline calibration gives $\dot{l}_*/l_* = -9.6 \times 10^{-6}$. Therefore, we almost have the balanced growth.

[19]The steady state is easy to find when the supply of labor is fixed. Equations (19) and (20) define $\hat{C}_\star$ and $\hat{k}^d_\star$ as a function of $\hat{K}_\star$ in a closed form. Substitute $\hat{k}^d_\star$ and $\hat{C}_\star$ into equation (18), which then becomes only a function of $\hat{K}_\star$. It can then be shown that the steady state is unique. Steady state values of a system with fixed labor supply can be used as starting values, when the steady state is numerically solved with elastic labor supply.

economy; and iii) How self-insurance works against shocks with permanent transition. That is, first we want to know if incomplete insurance against disability is an economically important issue. Second, we study how this welfare loss can be reduced. Finally, we explore in more detail how self-insurance works in this type of situation. This is interesting because self-insurance works quite well against moderate fluctuations in labor income, as shown by Aiyagari (1994).

4.1 Parameters selection and calibration

4.1.1 Parameters selection

We set parameters that are not directly in touch with the risk of disability or insurance against it, on the values that are traditionally used in growth models applied to the U.S. economy. Those values are as follows: $n = 0.01$, $\alpha = \frac{1}{3}$, $\rho = 0.04$, $g = 0.02$, $\delta = 0.05$ and $\phi = 1$. The last choice implies that Frisch elasticity is equal to 1, but we consider other values for Frisch elasticity as well. Given ϕ, we set γ, which then controls the level of labor supply, such that $l_\star = 0.33$.

4.1.2 The share of SSDI beneficiaries and the risk of disability

In 2009, the share of working age SSDI beneficiaries per labor force was 5.05%.[20] Since we do not have undeserving applicants in our model, we want to find out how many beneficiaries in SSDI are undeserving, which is measured by many studies. Benitez-Silva, Buchinsky, and Rust (2004) estimated that 20% of applicants who were accepted in the SSDI program were not actually disabled and Chena and van der Klaauw (2008) report similar values.[21] Hence, we set $\pi\lambda/\theta = 0.04$. Next, we find from the data that between 1980 and 2007, on average, an applicant was accepted into the SSDI program with a probability of 42%, that is, we set $\pi = 0.42$.

Given previously defined values, we calibrate λ such that $\pi = 0.42$, which yields $\lambda = 0.087$.

[20] Some target ratios used for calibration increase when the target for these ratios is their value at 2009. Data sources for which this calibration is based on are given in Web Appendix F, and the data is available on a data file associated with the paper.

[21] These estimates are almost the same as the estimate provided by Nagi (1969), who examined disability determinations with independent medical and social team. Nagi concluded that 19% of those initially awarded benefits were undeserving (see Benitez-Silva, Buchinsky, and Rust, 2004, Table 1).

This means that 8.7% of the working age population is disabled. This estimate is in line with self-reported disability status of different datasets, summarized by Bound and Burkhauser (1999, Table 2). Our estimate is close to the lower limit of reported values, but we only consider disabled persons who are permanently disabled and eligible for SSDI. The risk of disability can now be solved, though the arrival rate μ is almost impossible to calibrate directly from the data. Given the value of $\lambda = 0.087$, and when $n = 0.01$, we get $\mu = \frac{\lambda n}{1-\lambda}$=0.095%. This also implies that $\theta = 0.913$.[22]

4.1.3 Transfers received by the disabled population

We quantify the size of insurance against disability, which is coming from two different sources. First, we calibrate the replacement ratio η set by the government. The target for η is the benefits paid by the SSDI program as a percentage of GDP: 2009, the appropriate value was 0.67%.[23] In terms of the model, η is set such that $\frac{B_\star}{Y_\star} = 0.67\%$, which yields $\eta = 0.25$.[24] We consider η as a replacement ratio for a representative worker, when its low value for it is explained by the fact that people with low income more often receive SSDI benefits. However, here the calibration is done in a such way that the model captures the ratio of SSDI benefits per GDP.

Second, we calibrate private transfers to disabled persons, $\hat{b}$. The target for $\hat{b}$ is a decline in consumption that occurs due to onset of disability. Meyer and Mok (2008) estimate that individuals with a chronic and severe disability have experienced a 22% decline in food and housing consumption. Therefore, we calibrate $\hat{b} = 0.7$, when $\frac{c^d_\star}{c^e_\star} - 1 = -22\%$.[25] This implies that the employed

[22]Web Appendix D provides a detailed discussion of how different values for π matter for the values of λ and μ, which are critical here. Among the alternative measures for π, we have concluded that the one chosen here is the most appropriate.

[23]We have also reduced the value of benefits paid by 20%, since the number of people in SSDI were reduced by the same amount.

[24]Autor and Duggan (2003, Table 1) report the replacement ratio for non-elderly males at various percentiles of the wage distribution and ages for 1999. The replacement ratio varies significantly depending on age and earnings: The highest value is 74% and the lowest is 23%.

[25]Stephens (2001) estimates that consumption on food declines 5% due to onset of disability, and Low and Pistaferri (2010) report a 33% decrease in consumption for individuals who are disabled but do not receive SSDI benefits. However, they also find that those individuals who receive SSDI benefits actually increase their consumption due to onset of disability. To check the robustness of our results we also consider a calibration in which a decline in consumption is 10% due to onset of disability. That is, private insurance against disability is higher when $\hat{b} = 0.825$.

members of households give 5.2% of their income to the disabled members of households.

4.2 Welfare changes

To find out how much agents gain or lose under different types of policies, we must derive welfare changes. We measure welfare losses or gains in terms of consumption-equivalence variations. That is, we ask, how much must consumption increase or decrease in the current steady state so that a certain type of agent is indifferent to changes in the economy after the policy change?

We start by using a utilitarian welfare function, when we get a change in welfare where every agent is equally weighted. An alternative interpretation is that this is a welfare gain or loss of a newborn agent whose type is not known. The consumption-equivalent variation under a utilitarian welfare function is captured by ω^A. We also define conditional welfare changes, which defines a welfare change of certain type of agent. For employed agents we find ω^e, which captures expected change in welfare; for a representative disabled agent, we find ω^d. More details can be found in Web Appendix A.3.

4.3 The results

4.3.1 Insurance against disability in the United States

In simulations, we change the replacement ratio, η, and the value of private lump sum transfers, $\hat{b}$, to see the effects of different insurance schemes against disability. We compare the results of these experiments against the current system. Therefore, the results of simulations in Table 1 are reported as a percentage change of the values in column (1), which gives the values of our baseline calibration. However, interest rate, welfare changes and precautionary savings are given in absolute terms. Precautionary savings is defined as an additional capital stock, compared to the capital stock under complete markets. All variables are considered in their steady state values.

Column (1) shows the values of our baseline calibration, where $\eta = 0.25$ and $\hat{b} = 0.7$. The

Table 1: The results of simulations as a percentage change from column (1).†

	(1) Baseline calibration	(2) Complete markets	(3) Private insurance	(4) Pure self-insurance
	Variables at steady state values			
Output, $Y_\star$	0.530	1.34	0.50	8.24
Consumption, $C_\star$	0.401	1.42	0.50	2.31
Capital, $K_\star$	1.611	1.10	0.50	26.8
Labor, $N_\star$	0.304	1.64	0.50	0.02
Cons. for $\epsilon = 0$, $c^e_\star$	0.409	-0.57	0.50	9.29
Cons. for $\epsilon = 1$, $c^d_\star$	0.317	28.4	0.50	-92.2
After tax wage, $(1-\tau_\star)w_\star$	1.150	0.88	1.01	8.90
Interest rate, $r_\star \times 100\%$	5.973	6.00	5.97	4.37
Precautionary savings††	0.671	0	0.67	37.8
		Welfare changes		
Utilitarian, $\omega^A \times 100\%$		0.42	0.07	-12.7
For employed, $\omega^e \times 100\%$		-0.94	0.06	-1.24
For disabled, $\omega^d \times 100\%$		26.6	0.50	-89.2

† Interest rate, welfare changes and precautionary savings are given in absolute terms.
†† Precautionary savings are given as a percentage of aggregate wealth holdings.

calibration implies that precautionary savings can only explain 0.67% of net worth of households. The reason behind this result is that the low probability for disability induces a low expected reduction in consumption, even though the absolute decline in consumption due to onset of disability is quite large. This result is consistent with empirical results by Chandra and Samwick (2009), who estimated that the share of preretirement wealth attributable to the average disability risk is small.

Column (2) considers the costs coming from incomplete insurance against disability. There, we have set $\eta = 0$ and $\hat{b} = 1.0595$, which implies a complete private insurance against the risk of disability. The complete insurance against disability would remove the dis-saving of disabled persons, which leads to the higher stock of capital. The higher capital stock, in turn, increases demand for labor, and these two effects together generate a 1.4% increase in consumption and a 1.3% increase in output. However, the change in aggregate welfare is much smaller than the change in aggregate consumption, but it is still quite significant (0.42%).

The change in the aggregate welfare is a sum of two opposite effects: Workers' welfare declines

(-0.94%) as a result of decline in consumption on goods and leisure, but the reform would have a huge effect on the welfare of the representative disabled person (26.6%). In any case, we can conclude that the cost of incomplete insurance against disability causes a significant costs for the U.S. economy. Therefore, policy reforms that improve insurance against disability could also increase welfare and the performance of the aggregate economy quantitatively significantly.[26]

When the insurance markets are completed, the welfare of workers decreases, since they need to give more of their consumption to disabled persons than under the current system. However, there are levels of private transfers in which both workers and the disabled part of the population would be better off. In column (3), we have set $\eta = 0$ and $\hat{b} = 0.823$ when the welfare of both groups increases, compared to the current system.[27] Hence, this experiment shows that it is possible to shut down the public insurance system and replace it with a private one, which would lead to Pareto improvement. Disabled agents benefit from a better insurance, and employed agent welfare increases since distortions from proportional tax rate are removed. The crucial assumption, however, is that everyone can commit to these risk sharing contracts. The effects of public risk sharing in the presence of private insurance markets is discussed by Attanasio and Ríos-Rull (2000), Golosov and Tsyvinski (2007) and Krueger and Perri (2011), for example. These papers show that government insurance tends to crowd out private insurance quantitatively significantly in many situations. The result here supports the conclusions given by previous research that public insurance is not necessarily needed.

In column (4) we have set $\eta = 0$ and $\hat{b} = 0$, which implies that there is no private or public insurance against disability, when the only way to insure against disability is self-insurance by saving. Now, almost 38% of productive capital is held for precautionary reasons. This increase in capital stock causes an 8% increase in output and a 2% increase in aggregate consumption. However, moving from the current insurance system to pure self-insurance would reduce everyone's

[26]One example of this type of policy change is given by Golosov and Tsyvinski (2006), who showed in partial equilibrium that moving from the current system in the U.S. to an asset-tested disability insurance system produces an increase in lifetime utility which coincides with an increase in consumption of about 0.5%. Welfare improvement in Golosov and Tsyvinski (2006) is larger than here, since they also consider moral hazard problem. Moreover, the welfare gains suggested by Golosov and Tsyvinski can be much higher when general equilibrium effects are considered.

[27]We simply chose the level of private transfers such that the consumption shares of both groups increases equally.

welfare. Disabled persons' welfare would be reduced since their insurance income would decline significantly, and this also reduces the expected consumption of workers. Workers would achieve a higher consumption in the new steady state, but in the transition path they must postpone their consumption and work more, in order to build up precautionary savings, which reduces welfare. That is, pure self-insurance works poorly against permanent shocks, and some type of insurance is needed.

These results highlights two things: First, the risk of disability can cause a large precautionary saving motive affecting greatly to households' behavior. So, potentially it is an important source of income uncertainty and it should not been overlooked. Second, different insurance mechanism against disability has significant effects on the aggregate economy and these effects should be take into account when policy changes concerning SSDI are considered.

The robustness of the results is considered by changing the value for Frisch elasticity and examining a higher private insurance, when there is only a 10% decline in consumption due to onset of disability. Results are reported in Web Appendix E. When Frisch elasticity is equal to 3, we found that completing the insurance markets would increase output and aggregate consumption by about 2%. We also considered a value of 0.5 for Frisch elasticity, when completing the insurance markets would increase output and aggregate consumption by about 1%. Changes in welfare and values in other experiments are close to the values reported in Table 1. Moreover, higher private insurance causes a lower increase in welfare when insurance markets are completed (0.16%), but otherwise, changes are small compared to values in Table 1.

4.3.2 Self-insurance and risk sharing under shocks with permanent transition

Let us now study more carefully self-insurance against shocks with permanent transition. In the case of pure self-insurance, disabled agents need to finance their consumption purely with capital income, when additional transfer from workers definitely increases their welfare. However, workers' welfare may increase or decrease. A higher risk sharing lowers the consumption of workers, since they must give more of their income to disabled agents. There are aspects, however, that increase the welfare of workers as well: A higher consumption in disability state may increase the expected consumption of workers. Moreover, increased consumption in the disability state reduces precautionary saving motive, when workers can consume their precautionary savings on goods and leisure during the transition path. In Figure 1, we consider the quantitative importance of these two effects on the welfare of workers.

Figure 1 shows the change in the welfare of workers in a situation where the economy is moved from pure self-insurance to partial risk sharing. Risk sharing is measured with replacement ratios for labor income, which goes from 0 to 1. We consider both private risk sharing, when the insurance system is financed with lump sum transfers, and public insurance, when risk sharing is financed by distorting labor income tax. We have set up the model such that $\pi = 1$ and the replacement ratio is defined from after-tax income, when we can compare private and public insurance schemes against each other.

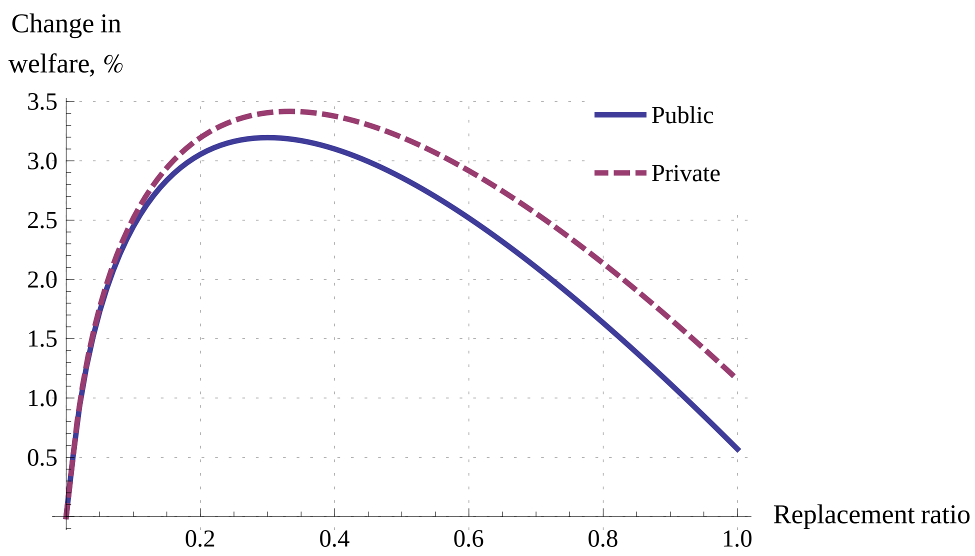

Figure 1: Changes in workers' welfare in consumption equivalent units, when the economy is moved from pure self-insurance to partial insurance, with different replacement ratios. Both private and public financed systems are considered.

Figure 1 clearly shows that risk sharing increases workers' expected welfare for all replacement ratios, and the size of welfare gains from risk sharing is large. Therefore, we can conclude that self-insurance works poorly against permanent shocks when workers would welcome a risk sharing mechanism. The result is consistent with Deaton (1991), who showed in partial equilibrium framework that the effectiveness of savings as an insurance mechanism against shocks to labor earnings declines as the persistence of these shocks rises. Here, the reason for the result is typical for Bewley-economies in general equilibrium: As more workers save for self-insurance, the interest rate lowers, which leads agents to quickly consume their savings in the disability state. Average consumption will therefore be lower in the disability state, which increases precautionary saving motive, and so forth. So, one of the driving forces behind poorly working self-insurance is that agents (here workers) do not take into account how their decisions influence prices (i.e., interest rate) as discussed earlier.

The second observation from Figure 1 is that with high replacement ratios, the public financed system produces a lower increase in welfare than the private one. This happens since distortionary labor income tax reduces the supply of labor, and the cost of this distortionary tax is quite large. Under private risk sharing, the maximum welfare gain for workers is 3.42%, which occurs with the replacement ratio of 0.39, whereas the public system produces a 3.20% increase in welfare at maximum, with a replacement ratio of 0.3. Hence, the private system produces 10.6% higher consumption for disabled agents than the public system, when workers' welfare is maximized. Put differently, at the situation in which workers' welfare is maximized, a private system produces 0.8% percentage points higher welfare than the public insurance system.[28] To summarize, we conclude that the above simulations showed a strong willingness for private risk sharing against permanent income shocks, since everyone involved would benefit.

5 Conclusions

The model in this paper extends the standard Ramsey model by a precautionary saving motive. That is, the members of the representative household face the constant probability of permanently losing their jobs throughout their lives and this uncertainty is only partially insured. Therefore, the model captures the uncertainty associated with the risk of disability, and could be used to

[28] A lower risk of disability causes smaller welfare gains from risk sharing, and with high replacement ratios, the change in workers' welfare turns out be negative. However, there still are significant gains from risk sharing. See Web Appendix E for a more detailed discussion.

study other types of "disaster" as well. That is, the model is suited to study the effects of uncertain events, which happen rarely, but their consequences are permanent.

We showed that households price taking behavior and endogenous prices lead to a situation in which the optimal decisions of disabled agents lead them to poverty. The reason behind this behavior is that disabled agents do not have income risk after they have been hit by the permanent disability shock and, therefore, the prevailing interest rate in the economy is "too low" for them. Put differently, the precautionary saving motive of workers, or incomplete insurance against disability, causes a negative externality, that is, the declining path for consumption in the disability state.

The quantitative analyzes of the model implied that precautionary savings against disability are small. But if there is no insurance against disability shocks, precautionary savings would explain about 38% of total wealth holdings of households. However, the large capital stock due to precautionary savings also implies that self-insurance works poorly, since it is based on capital income. Hence, there are large welfare gains from risk sharing against these type of shocks, and it seems natural that there is also strong private risk sharing against disability shocks. This also implies a possibility that the current SSDI can be replaced with a private insurance system such that everyone would benefit. However, poorly working self-insurance also implies that SSDI increases welfare, if the alternative is pure self-insurance.

The results in this paper implied two things: First, the risk of disability can cause a large precautionary saving motive affecting greatly to households' behavior. So, potentially it is an important source of income uncertainty. Second, different insurance mechanism against disability has significant effects on the aggregate economy and these effects should be take into account when policy changes concerning SSDI are considered. It seems that both issues are overlook by the current literature.

Since we assumed that agents can commit to risk sharing contracts, more research is needed to make these conclusion robust. Therefore, introducing a dynamic principal-agent problem into this framework would provide more robust results on the optimal public insurance against disability. That is, utilizing approaches introduced by Grochulski and Zhang (2011) and Williams

(2009) would allow us to study which type of the social insurance is optimal when agents cannot commit to risk sharing contracts. As well, more sophisticated modeling on household responses to disability shocks seems an interesting avenue for future research, as shown by Gallipoli and Turner (2011).

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